Number Theory Seminar

Integral Polynomial Pell Equations

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ABSTRACT: It is a very classical and old result that if d is a positive, non-square integer then the Pell equation $x^2 - dy^2 = 1$ has a solution with y non-zero. In the 1760's, Euler noticed that if d is of the form $d = n^2 - 1$, then one always has the solution: $(2n^2+1)^2 - (n^2+1)(2n)^2 = 1$.

One can view this identity as the first instance of a polynomial analogue of the Pell equation. In this talk we will discuss the polynomial analogue of the Pell equation, and show how existence of solutions correspond to arithmetic properties of associated abelian varieties. Using properties of elliptic curves, we will be able to classify all monic polynomials in $\mathbb{Z}[x]$ of degree at most 4 for which the Pell equation has non-trivial solutions in $\mathbb{Z}[x]$. Such results are wide open for degree at least 6.

Wednesday, March 19, 2014
2:40 - 4:00 PM
Room 527 Wachman Hall
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