Temple University Geometry Seminar

Louis Theran

University of Massachusetts

will speak on

Combinatorial genericity and minimal rigidity

ABSTRACT: A well studied geometric problem, with applications ranging from molecular structure determination to sensor networks, asks for the reconstruction of a set P of n unknown points from a finite set of pairwise distances (up to Euclidean isometries). We are concerned here with a related problem: which sets of distances are minimal with the property that they allow for the reconstruction of P, up to a finite set of possibilities? In the planar case, the answer is known generically via the landmark Maxwell–Laman Theorem from Rigidity Theory, and it leads to a combinatorial answer: the underlying structure of such a generic minimal collection of distances is a minimally rigid (or Laman) graph, for which very efficient combinatorial decision algorithms exist. For non-generic cases the situation appears to be dramatically different, with the best known algorithms relying on exponential-time Gröbner base methods, and some specific instances known to be NP–hard. Understanding what makes a point set generic emerges as an intriguing geometric question with practical algorithmic consequences.

Several definitions (some but not all equivalent) of genericity appear in the rigidity literature, and they have either a measure theoretic, topologic or algebraic–geometric flavor. Some generic point sets appear to be highly degenerate, and still turn out to be generic. All existing proofs of Laman's Theorem make use at some point of one or another of these geometric genericity assumptions.

In this talk, I will describe the first purely combinatorial proof of Laman's theorem, together with some interesting consequences. Genericity is characterized in terms of a certain determinant being not identically-zero as a formal polynomial. We relate its monomial expansion to certain colorings and orientations of the graph and show that these terms cannot all cancel exactly when the underlying graph is Laman. As a surprising consequence, genericity emerges as a purely combinatorial concept. Joint work with Ileana Streinu.

Tuesday, 24 February 2009
Lecture at 2:40 pm
Coffee, tea, and refreshments from 3:30–5 pm
Room 617, Wachman Building
Department of Mathematics