TEMPLE UNIVERSITY MATHEMATICS COLLOQUIUM

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will speak on

Modeling Tug-of-War games with noise using the p–Laplacian

ABSTRACT: Consider the following game played by two players, whom we call Henry and Theresa. A token is placed in an initial position of a board (graph). For every vertex x there is a predetermined set of successors $S(x)$ in the board. We are also given two non-negative numbers α and β such that $\alpha + \beta = 1$. With probability α Henry and Theresa play a step of a tug-of-war game. They flip an unbiased coin. If the result is heads, Henry gets to move the token from x to anywhere in $S(x)$. If the result is tails, Theresa moves instead to any point in $S(x)$ of her choice. With probability β the token moves from x to a random point in $S(x)$. This is the noisy part of the game. We are also given a pay-off function F defined on the boundary of the board. The game ends when one of the players places a token at a boundary point y. Then Henry pays Theresa \$$F(y)$.

   Imagine that we play this game many times over. What is the expected amount of money Theresa will get? Or what is the expected amount of money Henry will have to pay? Of course, the answer will depend on how well they play the game. When is Henry's turn to play, he will try to approach points on the boundary where F is as small (negative) as possible, while Theresa will try to approach points where F is as large (positive) as possible. It turns out that if Henry and Theresa play according to optimal strategies, the amount of money they can expect to get (or pay) starting at x is the solution to a difference equation that is the discrete analogue of the p–Laplace equation

$$\sum_{i,j}^{n} \left\{ \delta_{ij} + (p-2)\frac{u_i u_j}{|\nabla u|^2} \right\} u_{ij} = 0.$$

   This is joint work with Mikko Parviainen (Helsinki) and Julio D. Rossi (Buenos Aires).

MONDAY, 22 FEBRUARY 2010
LECTURE AT 4:00 PM
COFFEE, TEA, AND REFRESHMENTS FROM 3-5 PM
ROOM 617, WACHMAN BUILDING
DEPARTMENT OF MATHEMATICS