Temple University Mathematics Colloquium

Herman Gluck

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will speak on

Optimality of Hopf fibrations and Hopf vector fields

ABSTRACT: In 1931, Heinz Hopf, then in the early years of his mathematical career, introduced a simple quadratic polynomial map from the unit 3-sphere to the unit 2-sphere, so that the point-inverse-images were mutually parallel great circles. This provided the first example of a homotopically nontrivial dimension-lowering map between spheres, and signaled the birth of homotopy theory.

This example was soon joined by higher-dimensional cousins, all now called "Hopf fibrations", in which a round sphere is filled by a family of mutually parallel great subspheres.

We will show that in every case, the projection map from the round sphere to the base space of the fibration is, up to isometries of domain and range, the unique Lipschitz constant minimizer in its homotopy class.

Moreover, given a Hopf fibration of a round sphere (of dimension $3, 5, 7, \ldots$) by parallel great circles, we view the unit vector field tangent to the circles as a cross-section of the unit tangent bundle of the sphere, and show that it is, up to isometries of domain and range, the unique Lipschitz constant minimizer in its homotopy class.

Previous attempts to find a mathematical sense in which Hopf fibrations and vector fields are optimal have met with limited success.

This is joint work with Dennis DeTurck and Peter Storm.

Monday, 23 November 2009
Lecture at 4:00 pm
Coffee, tea, and refreshments from 3-5 pm
Room 617, Wachman Building
Department of Mathematics