Temple University Mathematics Colloquium

Peter Shalen

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will speak on

Homology and volume of hyperbolic 3-orbifolds, and enumeration of arithmetic groups

ABSTRACT:

An arithmetic lattice in a quaternion algebra is roughly analogous to the group of units in the integers of a number field. Remarkably, certain arithmetic lattices can be studied via the geometry and topology of 3-manifolds, or more generally of objects called 3-orbifolds. The manifolds (or orbifolds) associated with arithmetic lattices are hyperbolic manifolds, locally modeled on the 3-dimensional non-Euclidean geometry of Gauss, Bolyai and Lobachevsky (or quotients of the latter by finite groups). In particular, each of these manifolds has a well-defined volume.

A theorem of Borel's asserts that for any positive real number V, there are at most finitely many arithmetic lattices of covolume at most V. Determining all of these for a given V is algorithmically possible for a given V thanks to work by Chinburg and Friedman, but appears to be impractical except for very small values. It turns out that the difficulty in the computation for a larger value of V can be dealt with if one can find a good bound on $\dim H_1(O, \mathbb{Z}/2\mathbb{Z})$. In the case of a hyperbolic 3-manifold M, not necessarily arithmetic, joint work of mine with Marc Culler and others gives good bounds on the dimension of $H_1(M, \mathbb{Z}/2\mathbb{Z})$ with a suitable bound on the volume of M. I will discuss a program for finding analogous results for hyperbolic 3-orbifolds, partial results obtained so far, and prospects for applying results of this kind to the enumeration of arithmetic lattices.

Monday, October 27
Lecture at 4:00 pm
Coffee, tea, and refreshments from 3:40 pm
Room 617, Wachman Hall
Department of Mathematics