TEMPLE UNIVERSITY MATHEMATICS COLLOQUIUM

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will speak on

Homogenization in problems with non-separated scales

ABSTRACT: Classical Homogenization Theory deals with mathematical models of strongly inhomogeneous media composed of materials with vastly different properties described by PDEs with rapidly oscillating coefficients of the form $A(x/\epsilon)$, $\epsilon \to 0$. The goal is to replace (approximate) this problem by a homogenized (simpler) PDE with slowly varying coefficients that do not depend on the small parameter ϵ. The original problem has two scales: fine $O(\epsilon)$ and coarse $O(1)$, whereas the homogenized problem has only coarse scale.

The homogenization of PDEs with periodic or ergodic coefficients and well separated scales is now well understood. In a joint work with H. Owhadi (Caltech) we consider the most general case of arbitrary bounded coefficients. Specifically, we study divergence-form scalar elliptic equations and vectorial equations for elasticity with arbitrarily rough ($L^\infty(\Omega)$, $\Omega \subset \mathcal{R}^d$) coefficients $a(x)$. For these problems we establish two finite dimensional approximations of solutions, which we refer to as homogenization approximations:

- an approximation by a global basis with an explicit and *optimal* error constant independent of the contrast and regularity of the coefficients.
- an approximation with a minimal amount of pre-computation with both global and *local* bases.

We define the flux norm (which is equivalent to the usual H^1-norm) as the L^2-norm of the potential part of the fluxes of solutions. We show that in this norm, the error associated with approximating (in a properly defined finite-dimensional space) the solution to the aforementioned PDE with rough coefficients is equal to the error associated with approximating the solution to the same type of PDE with smooth coefficients in a standard space (e.g., piecewise polynomial). We refer to this property as the *transfer property.*

The results described above are obtained as an application of the transfer property as well as a new class of elliptic inequalities which we conjecture. These inequalities play the same role in our approach as the div-curl lemma in classical homogenization.

MONDAY, 5 OCTOBER 2009
LECTURE AT 4:00 PM
COFFEE, TEA, AND REFRESHMENTS FROM 3-5 PM
ROOM 617, WACHMAN BUILDING
DEPARTMENT OF MATHEMATICS