TEMPLE UNIVERSITY
Department of Mathematics

Applied Mathematics and Scientific Computing Seminar

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Imposing Jump Conditions on Nonconforming Interfaces via Least Squares Minimization

by Rodolfo Ruben Rosales
Massachusetts Institute of Technology

Abstract. We introduce a method to impose jump conditions on interfaces that are not aligned with a computational grid. In particular, we discuss the method in the context of solving the Poisson equation with prescribed jump discontinuities across an internal interface, using the Correction Function Method (CFM). The CFM offers a general framework to solve equations in the presence of discontinuities, to high order of accuracy, while using a compact discretization stencil. A key concept behind the CFM is enforcing the jump conditions in a least squares sense. This approach requires computing integrals over pieces of the interface, which becomes challenging when only an implicit representation of the interface is available (e.g., the zero contour of a level set function), especially in 3D. The technique is based on a least squares procedure that depends only on integrals over sections of the interface that are amenable to numerical quadrature after appropriate coordinate transformations.

Computations that involve moving interfaces across which the variables (and their gradients) jump are very important in practice. In particular, embedded interface methods, which do not require re-meshing as the interfaces move, are very attractive, but hard to push into high orders. Two well known examples are the Immersed Boundary Method (IBM), and the Immersed Interface Method (IIM). In this talk I will introduce a close cousin to these methods, using the example of the Poisson equation.

The Poisson equation with jumps in function value and normal derivative across an interface is of central importance in Computational Fluid Dynamics. The basic idea of the CFM is similar to the Ghost Fluid Method (GFM), which relies on corrections applied on nodes located across the interface for discretization stencils that straddle the interface. If the corrections are solution-independent, they can be moved to the right hand side (RHS) of the equations, producing a problem with the same linear system as if there were no jumps, but different RHS. If not, the result is a modified linear system which has the same sparsity pattern as the standard discretization. However, achieving high accuracy is not simple with the "standard" approaches used to compute the GFM correction terms.

In the CFM the corrections are computed via a "correction function", defined in a narrow band around the interface via a pde with with appropriate boundary conditions. This pde can, in principle, be solved to any desired order of accuracy. For example, a solution compatible with 4-th order accuracy can be obtained by representing the correction function in terms of Hermite interpolants (bi-cubics in 2D), followed by an appropriate least square minimization. Higher order accuracy follows, in principle, by using higher order interpolants, with the same approach.