TEMPLE UNIVERSITY
Department of Mathematics

Applied Mathematics and Scientific Computing Seminar

Room 617 Wachman Hall

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The Radial Basis Function (RBF) Method; An Efficient Meshless Approach to the Solution of PDEs

by Manuel Kindelan
Universidad Carlos III de Madrid

Abstract.

Radial Basis Functions (RBF) originate as a very efficient technique for the interpolation of multi-dimensional scattered data. Later, it became popular as a truly meshfree method for the solution of partial differential equations (PDEs) on irregular domains. The main advantages of the method are ease of programming and potential spectral accuracy, but its main drawback is ill-conditioning of the resulting linear system. To overcome this drawback a local version of the method was later proposed. The idea of the local RBF method, is to sacrifice the spectral accuracy inherent to the global method, in order to have a sparse better conditioned linear system capable of solving large multidimensional PDEs. The method is often denoted RBF-FD (Radial Basis Function-Finite Difference method) and it works very similarly to the finite difference (FD) method: differential operators at a given node are approximated as a weighted sum of the values of the sought function at some surrounding nodes. However, while in the FD method the unknown weights are computed using polynomial interpolation, in the local RBF method they are computed by fitting an RBF interpolant through a stencil of neighboring nodes. Most of the RBFs used in practical applications depend on a shape parameter, and there is much experimental evidence showing that the accuracy of the solution to a PDE strongly depends on the value of this parameter. In the first part of the talk I will present an overview of the global and local RBF method. In the second part I will describe some recent work focused on the determination of the optimal value of the shape parameter and how to compute it efficiently. The method is based on analytical approximations to the local truncation error. Two different strategies will be analyzed. The first one computes a constant value of the shape parameter which minimizes the norm of the error in the whole domain. The second computes a set of variable shape parameters which minimize the local truncation error in every node. Both strategies result in several orders of magnitude increase in accuracy with respect to standard FD.