TEMPLE UNIVERSITY
Department of Mathematics

Applied Mathematics and Scientific Computing Seminar

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Particle Methods for Nonlinear Time-Dependent PDEs

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Abstract.

In recent years, particle methods have become one of the most useful and widespread tools for approximating solutions of partial differential equations in a variety of fields. In these methods, the solution is sought as a linear combination of Dirac distributions, whose positions and coefficients represent locations and weights of the particles, respectively. The solution is then found by following the time evolution of the locations and the weights of the particles according to a system of ODEs, obtained by considering a weak formulation of the problem. The main advantage of the particle methods is their low numerical diffusion that allows to capture a variety of nonlinear waves with a high resolution. Even though the most "natural" application of the particle methods is linear transport equations, over the years, the range of these methods has been extended for approximating solutions of nonlinear equations including degenerate parabolic, convection-diffusion and dispersive equations.

In this talk, I will review different aspects of a practical implementation of particle methods such as recovering an approximate solution from the particle distribution and investigation of various particle redistribution algorithms. I will also present new numerical techniques for nonlinear PDEs, with particular reference to problems that admit nonsmooth (discontinuous) solutions and on problems that involve multiple scales, and therefore, are difficult to solve by traditional finite-difference methods. I will demonstrate the performance of the new methods on a number of numerical examples, among which are models of transport of pollutant in shallow water, pressureless gas dynamics, and models describing the interaction dynamics of singular wave fronts.