TEMPLE UNIVERSITY
Department of Mathematics

Applied Mathematics and Scientific Computing Seminar

Wednesday, 3 April 2013, 4:00 p.m.
Room 617 Wachman Hall

(refreshments and social at 3:45 p.m)

A generalization of conjugate points for a broader range of elastic rod problems

by Robert Manning
Haverford College

Abstract. The buckling of an elastic rod is a classic problem in the calculus of variations; in other words, it is a minimization problem over the domain of a set of functions. Equilibrium rod configurations are determined by solving the Euler-Lagrange equations, the function-space analogue of the familiar Calculus I equation $f'(x) = 0$ for finding critical points of a function f.

The classic calculus of variations theory also includes a method for determining the stability of a critical point (in the sense of whether it minimizes the energy) via the idea of conjugate points (summarized in short by the theorem "stable if and only if no conjugate point"). However, the classic methods do not make it clear how to extend this idea of conjugate points to a wide range of problems, such as rod buckling that accounts for self-contact or contact with an obstacle. I will present a reformulation of conjugate point theory that shows the connection to the analogous finite-dimensional theory (specifically, to the spectrum of the Hessian). This theory allows a relatively easy generalization of conjugate points to many problems, including the examples cited above, and also reveals that sometimes (such as with Neumann-Neumann boundary conditions), "stable if and only if no conjugate point" is false.