TEMPLE UNIVERSITY
Department of Mathematics

Applied Mathematics and Scientific Computing Seminar

Wednesday, 11 November 2015, 4:00 p.m.
Room 617 Wachman Hall

(refreshments and social at 3:45 p.m)

Numerical Approximation of Coefficients of Belyĭ Maps

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Abstract. In 1984, Alexander Grothendieck, inspired by a result of Gennadiĭ Belyĭ from 1979, constructed a finite, connected planar bipartite graph via rational functions $\beta : \mathbb{P}^1(\mathbb{C}) \to \mathbb{P}^1(\mathbb{C})$ with critical values $\{0, 1, \infty\}$ by looking at the inverse image of the triangle formed by these three points. He called such graphs Dessins d'Enfants. Conversely, Riemann's Existence Theorem implies that every finite, connected planar graph arises in this way – once one labels the vertices as "0", the midpoints of the edges as "1", and the midpoints of the faces as "∞".

The difficulty arises in explicitly constructing such a Belyĭ map β from any given planar graph. We may form a valency list by considering the number of edges surrounding each vertex and each face; this forces algebraic conditions on the coefficients of the desired Belyĭ map. Hence the construction of a Belyĭ map can be reduced to the computation of roots of a system of nonlinear equations. In this talk, we reformulate the problem of finding these roots into an unconstrained optimization problem. We implement Newton's method and a limited-memory BFGS method to approximate these coefficients. Preliminary results are presented and possible directions are discussed.