

TEMPLE UNIVERSITY  
Department of Mathematics

**Applied Mathematics and  
Scientific Computing Seminar**

Room 617 Wachman Hall

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*Multiscale Analysis and Modeling of Ferromagnetic Shape  
Memory Composites*

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Shape memory materials are alloys that are able to revert back to their original shape at a temperature above certain critical temperature, no matter how they are deformed at a temperature below the critical temperature. Ferromagnetic Shape Memory (FSM) alloys are both ferromagnetic and have shape-memory. Deformations of FSM alloys induce magnetic fields and vice versa. There is a great deal of industrial interest in using FSM alloys to generate large forces and movements by applying magnetic fields since they exhibit the largest known work output per unit volume of the material.

We calculate the effective properties of a FSM composite in the cases of the dilute limit and finite volume fractions. The composite consists of identical FSM particles, surrounded by an elastic matrix. In the case of the dilute limit we show that the energy minimization problem for the composite can be cast as a quadratic programming problem by using results from a Constrained theory and from the Eshelby inclusion problem in linear elasticity. In the case of finite volume fraction, we assume the composite has periodic structure and the embedded FSM particles are much smaller than the overall composite body. Using multiscale methods, we again manage to cast the minimization problem as a quadratic programming problem, provided some special microstructures exist.

The existence problem for these special microstructures is related to variational inequalities. To visualize, one may imagine pushing an elastic membrane down on an obstacle. The desired special microstructure is formed exactly where the membrane contacts the obstacle. We call these special microstructures E-inclusions. E-inclusions apparently enjoy many interesting properties with respect to homogenization and energy minimization problems. In particular, we use them to give new results on a) optimal bounds of the effective moduli of two-phase composites, b) energy-minimizing microstructures; and c) the characterization of the G-closure of two well-ordered conductors.