TEMPLE UNIVERSITY
Department of Mathematics

Applied Mathematics and Scientific Computing Seminar

Room 617 Wachman Hall

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Optimality of Methods Minimizing an Energy Norm

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Abstract. Two-level Schwarz preconditioners are known to be optimal in the sense that convergence bounds for the preconditioned problem exist which are independent of the mesh and the number of subdomains. These bounds are based on some kind of energy norm. On the other hand, if the iterative method used minimizes the Euclidean norm of the residual, this optimality may be lost; see [X.-C. Cai and J. Zou, *Numer. Linear Algebra Appl.*, 9:379–397, 2002]. In this paper, iterative methods are presented which minimize the same energy norm in which the optimal Schwarz bounds are derived, thus maintaining the Schwarz optimality. The problems we have in mind are of the following form:

$$b(u,v) = f(v) \quad \text{for all} \quad v \in V,$$

where

$$\begin{aligned} b(u,v) &= a(u,v) + s(u,v) + c(u,v), \\ a(u,v) &= \int_\Omega \nabla u \cdot \nabla v \, dx, \end{aligned}$$

$$s(u,v) = \int_\Omega (b \cdot \nabla u)v \;+\; (\nabla \cdot bu)v \, dx, \quad b \in \mathbb{R}^d,$$

$$c(u,v) = \int_\Omega c\, uv \, dx, \qquad \text{and} \qquad f(v) = \int_\Omega f\, v \, dx.$$

(Joint work with Marcus Sarkis, Instituto Nacional de Matemática Pura e Aplicada, Rio de Janeiro, Brazil, and Worcester Polytechnic Institute, Worcester, MA)