Temple University
Department of Mathematics

Applied Mathematics and Scientific Computing Seminar

Room 617 Wachman Hall

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Local Minimizers in Calculus of Variations. Part I

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Abstract. Consider the variational problem of minimizing the functional

$$I(\mathbf{y}) = \int_\Omega W(\nabla \mathbf{y}(\mathbf{x}))d\mathbf{x}$$

where, $\Omega \subset \mathbb{R}^d$ an open bounded set, $W : \mathbb{R}^{m\times d} \to \mathbb{R}$, a continuous function, and $\mathbf{y} \in \mathcal{C}$. Here $\mathcal{C} = \{\mathbf{y} \in W^{1,\infty}(\Omega;\mathbb{R}^m) : \mathbf{y}|_{\partial\Omega_1} = \mathbf{y}_0\}$ is the set of competing maps for some $\mathbf{y}_0 \in W^{1,\infty}(\Omega;\mathbb{R}^m)$ and $\partial\Omega_1 \subset \partial\Omega$

The notion of local mimimizers depends on the topology on $\mathcal{C}$. We will discuss weak and strong local minimizers corresponding to the $W^{1,\infty}$, and L^∞ topologies on $\mathcal{C}$ respectively. The main question in Calculus of Variations is to formulate necessary and sufficient conditions a function $\mathbf{y}$ must satisfy to be a strong (weak) local minimizer of I. The classical necessary conditions solving Euler-Lagrange equations and nonnegativity of the second variation will be discussed. The notion of quasiconvexity (which coincides with convexity when $m = 1$ or $d = 1$) as a necessary condition for strong local minimizers will be introduced, and a sufficiency theorem will be proved.