TEMPLE UNIVERSITY
Department of Mathematics

Applied Mathematics and Scientific Computing Seminar

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Optimized Domain Decomposition Methods, II

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A domain decomposition method is a way of solving boundary value problems (BVP) on large domains by iteratively solving similar problems on smaller subdomains. There is no unique choice for the BVP on the subdomains. In the first of two talks, I will introduce all basic notions which have been worked out chiefly through the 1980s and 1990s. In the second talk, I will discuss the latest methods for analyzing a class of algorithms known as Optimized Schwarz Methods. Consider the problem to find $u \in H_0^1(\Omega)$ such that $\Delta u = u_{xx} + u_{yy} = f$ for a given $f \in L^2(\Omega)$. The Schwarz iteration for given u_0^0, u_1^0 is

$$\begin{cases} \Delta u_j^{(k)} = f & \text{in } \Omega_j, \\ u_j^{(k)} = 0 & \text{on } \partial\Omega_j \cap \partial\Omega, \\ u_j^{(k)} = u_{1-j}^{(k-1)} & \text{on } \partial\Omega_j \cap \Omega_{1-j}; \end{cases}$$

where $\Omega = \Omega_0 \cup \Omega_1$, $j = 0, 1$ and $k = 1, 2, ...$ Lions showed in 1988 that $u_j^{(k)} \to u|_{\Omega_j}$ as $k \to \infty$. Optimized algorithms replace the third line by $D_\nu u_j^{(k)} + \alpha u_j^{(k)} = D_\nu u_{1-j}^{(k-1)} + \alpha u_{1-j}^{(k-1)}$ (for example), where D_ν denotes the derivative normal to the boundary, and $\alpha \in \mathbb{C}$ is a "relaxation parameter". The problem becomes that of choosing the best possible α.