TEMPLE UNIVERSITY
Department of Mathematics

Applied Mathematics and Scientific Computing Seminar

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Algebraic methods in materials science–a case study

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A central problem in materials science is the study of how physical properties of composites such as conductivity and elasticity depend on the properties of their constituents. In general, these properties depend strongly on the microstructure; however, in exceptional situations, there exist exact formulas for the effective properties of composites which are microstructure-independent. These formulas represent fundamental physical invariances.

The classical literature on exact relations is very extensive, but the methods used have been heavily dependent on the details of the physical context. Thus, individual exact relations were discovered for specific physical problems, but the results did not generalize to other settings. Grabovsky and Grabovsky-Milton-Sage have developed an abstract theory of exact relations, which has reduced the search for them to a purely algebraic problem involving group representation theory. This general theory has proved to be enormously powerful. Indeed, it has led to complete lists of all rotationally invariant exact relations for three-dimensional thermopiezoelectric composites, which include all exact relations for elasticity, thermoelasticity, and piezoelectricity as special cases.

The fact that algebraic methods have been the key to this progress has come as quite a surprise. It is equally surprising that the problems raised in representation theory have been completely new, and in fact, the advances in materials science have been accompanied by advances in pure representation theory. In this talk, we will discuss some of these novel algebraic problems and their significance in the study of exact relations.