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FLAT POINTS IN ZERO SETS OF HARMONIC POLYNOMIALS:
INTERACTION BETWEEN ANALYSIS AND GEOMETRY

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Abstract: In this talk, I want to describe a theorem about the local geometry of zero sets of harmonic polynomials, which connects analytic and geometric notions of 'regular points'. While analytic regularity of a zero set at a point is indicated by the non-vanishing of the Jacobian of a defining function, geometric regularity of a zero set is displayed by the existence of a tangent plane to the set. Alternatively, one may equate geometric regularity with existence of arbitrarily good approximations of the set by hyperplanes at smalls scales. It turns out that for zero sets of harmonic polynomials these two types of regularity, analytic and geometric, coincide. Moreover, the failure of the zero set of a harmonic polynomial to admit good approximations by hyperplanes at its singularities can be quantified. Simple examples show that this feature of harmonic polynomials does not hold for the zero sets of generic polynomials. If time permits, then I will also describe an application of this result to a free boundary problem for harmonic measure.