TEMPLE UNIVERSITY

Department of Mathematics

Analysis Seminar

Room 617 Wachman Hall

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Quantum Painleve II equation (QPII) and random matrix beta ensembles.

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Abstract: The six classical Painleve equations found numerous applications in different branches of science. E.g. Painleve II (PII) is related to the celebrated Tracy-Widom distributions of random matrix theory and their universality. Painleve ODEs can be seen as classical one-particle dynamical systems with time-dependent Hamiltonians. The Quantum Painleve equations (QPEs) are linear Fokker-Planck or non-stationary Schroedinger PDEs in two independent variables ("time" and "space") with spatial operators being quantized Painleve Hamiltonians. QPEs are satisfied by certain eigenvalue probabilities of random matrix beta ensembles (or probabilities of Coulomb gas particle positions restricted to a line). E.g. QPII describes the soft edge limit of beta ensembles while QPIII does so for the hard edge.

We construct classical nonlinear integrable structure associated with QPII, more explicit for even integer beta. The nonlinear PDEs tied with QPII allow one to gain more information about the QPII solutions. The corresponding probability distributions explicitly depend on Hastings-McLeod solution of PII in all known cases and conjecturally for all values of beta.

If time permits, I plan to discuss open problems related to QPEs and their generalizations relevant in various applications including multivariate statistics, Coulomb gases in the plane, stochastic Loewner evolutions (SLEs), quantum Hall effect, black hole physics, string theory and others.