TEMPLE UNIVERSITY

Department of Mathematics

Analysis Seminar

Zoom meeting

Monday November 30, 2020, 2:30 p.m.

Degenerate Elliptic Boundary Value Problems with Non-smooth Coefficients

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Abstract: Let X be a manifold with boundary and bounded geometry. On X we consider a uniformly strongly elliptic second order operator A that locally is of the form

$$A = -\sum_{j,k} a_{jk}\partial_{x_j}\partial_{x_k} + \sum_j b_j\partial_{x_j} + c.$$

A is endowed with a boundary operator T of the form

$$T = \varphi_0\gamma_0 + \varphi_1\gamma_1,$$

where γ_0 and γ_1 denote the evaluation of a function and its exterior normal derivative, respectively, at the boundary, and φ_0, φ_1 are non-negative C_b^∞ functions on the boundary with $\varphi_0+\varphi_1 \geq c_0 > 0$. This problem is not elliptic in the sense of Lopatinskij and Shapiro, unless either $\varphi_1 \neq 0$ everywhere or $\varphi_1 = 0$ everywhere.

We show that the realization A_T of A in $L^p(\Omega)$ has a bounded H^∞-calculus of arbitrarily small angle whenever the a_{jk} are Hölder continuous and b_j as well as c are L^∞.

For the proof we first treat the operator with smooth coefficients on $\mathbb{R}^n_+$. Here we rely on an extension of Boutet de Monvel's calculus to operator-valued symbols of Hörmander type $(1,\delta)$. We then use H^∞-perturbation techniques in order to treat the non-smooth case.

The existence of a bounded H^∞-calculus allows us to apply maximal regularity techniques. We show how a theorem of Clément and Li can be used to establish the existence of a short time solution to the porous medium equation on X with boundary condition T.

(Joint work with Thorben Krietenstein, Hannover)