TEMPLE UNIVERSITY

Department of Mathematics

Analysis Seminar

Wachman 617

Monday, September 12, 2022, 2:30 p.m.

L^∞-estimates in optimal transport for non quadratic costs

by Cristian E. Gutiérrez

Temple University

Abstract: This research originates from recent results by M. Goldman and F. Otto concerning regularity of optimal transport maps for the quadratic cost. We consider cost functions having the form $c(x,y) = h(x-y)$, where h is positively homogeneous of degree $p > 1$ and $h \in C^2(\mathbb{R}^n \setminus \{0\})$. A mapping $T : \mathbb{R}^n \to \mathbb{R}^n$ is c-monotone if $c(Tx,x) + c(Ty,y) \leq c(Tx,y) + c(Ty,x)$. Using Green's representation formulas, if T is c-monotone, we prove local L^∞-estimates of $Tx - x$ in terms of L^p-averages of $Tx - x$. From this we deduce estimates for the interpolating maps between T and Id, and when T is optimal, L^∞-estimates of $T^{-1}x - x$. As a consequence of the technique, we also obtain a.e. differentiability of monotone maps. This is joint work with Annamaria Montanari (Bologna) to appear in Calculus of Variations and PDEs.