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Hardy spaces: classical, extensions and applications

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The Hardy spaces theory H^p had its origins in the extraordinary discoveries seventy or eighty years ago by G. H. Hardy, J. E. Littlewood, I. I. Privalov, F. and M. Riesz to cite only the most known. Fatou proved in 1906 [Fa] that any bounded and holomorphic function f in the $\Delta = \{z \in \mathbb{C} : |z| < 1\}$ has a nontangencial limit a.e. that can not vanish in any arc of positive measure of $\partial\Delta$ unless f be identically zero. Later, in 1915, Hardy [Ha] began the Hardy spaces theory $H^p(\Delta)$ showing that the logarithm of the $L^p[-\pi, \pi]$ norm of $\theta \to f(re^{i\theta})$ is a convex function of $\ln r$, $0 < r < 1$. The Chapters 7 and 14 of Zygmund's treatise about Trigonometric Series [Z] exhibit in a unified way the H^p's in the context of holomorphic functions of one complex variable. In the late 1950's the development of the methods of real variable used some years ago by Calderón-Zygmund school in Chicago —that allow proving classical results without using holomorphic function theory such as the Hilbert's transform continuity— gives the possibility to the real treatment of the H^p spaces in several variables, initiated by E. Stein and G. Weiss [SW] with their maximal characterization, completed by duality between H^1 and BMO due to C. Fefferman and E. Stein, [FS] and by the atomic characterization formulated and proved by R. Coifman (in one dimension) although it was presented in an implicit and primitive way in the duality result.

The aim of this mini course is to present historical and recent developments of the Hardy spaces theory and applications to PDE's, such as: 1) characterise the boundary values of solutions of (system) holomorphic vector field(s) as in [HH, HdosS]; 2) Hardy-Sobolev atomic decompositions and application to obtain a "new" proof of the Div-Curl lemma as in [HA]; and 3) Div-curl type estimates for elliptic systems of complex vector fields as in [HHP].

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