Boundary value problems for elliptic operators

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Abstract: This talk reflects work in progress with T. Krainer directed at studying elliptic boundary value problems in a context well beyond the classical set-up.

Let $\mathcal{M}$ be a smooth noncompact manifold, or just an open set in some euclidean space. Let μ be a smooth density on $\mathcal{M}$ and P an elliptic linear partial differential operator on $\mathcal{M}$ with smooth coefficients.

The operator P, initially viewed as an unbounded operator

$$C_c^\infty(\mathcal{M}) \subset L^2(\mathcal{M}, \mu) \to L^2(\mathcal{M}, \mu)$$

has two (well known) canonical extensions as a closed operator, namely its closure starting with domain $C_c^\infty(\mathcal{M})$ and the maximal extension, the extension whose domain is $\mathcal{D}_{\max} = \{u \in L^2 : Pu \in L^2\}$.

I will discuss briefly the meaning of traces (or boundary values) for elements of $\mathcal{D}_{\max}$ in general, then particularize the problem to the case of so-called elliptic wedge operators, a large class which includes regular elliptic operators on compact manifolds with boundary, or open bounded sets in euclidean space with smooth boundary, as well as certain operators on manifolds with simple geometric singularities. For the class in question I will describe in some detail how to construct a vector bundle whose sections are the full traces (to certain order) of elements in the maximal domain of the operator.