UNIFORM RECTIFIABILITY AND HARMONIC MEASURE ON 1-SIDED NTA DOMAINS José María Martell

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Abstract

A connected open set is a 1-sided NTA domain if it satisfies interior (but not necessarily exterior) Corkscrew and Harnack Chain conditions. These two conditions are, respectively, quantitative or scaleinvariant versions of the openness and path-connectedness. We establish that for 1-sided NTA domains $\Omega \subset \mathbb{R}^{n+1}$, $n \geq 2$, with Ahlfors-David regular boundary, the following statements are equivalent: (1) $\partial\Omega$ is Uniformly Rectifiable; (2) harmonic measure is absolutely continuous with respect to surface measure on $\partial\Omega$, with scale invariant higher integrability of the Poisson kernel; (3) the Riesz transform is bounded in $L^2(\partial\Omega)$.

The implication $(1) \Longrightarrow (2)$ is a higher dimensional, scale-invariant version of the classical theorem of F. and M. Riesz, which established absolute continuity of harmonic measure with respect to arc length measure, for a simply connected domain in the complex plane with a rectifiable boundary. The implication $(3) \Longrightarrow (1)$ is a partial solution to a conjecture posed by David and Semmes, which has been recently solved by Nazarov, Tolsa and Volberg.

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