

Quasispheres and bi-Lipschitz parameterizations

Matthew Badger
Stony Brook University

Abstract: A quasisphere is the image of the unit sphere under a global quasiconformal map. If the quasiconformal dilatation of the defining map restricted to a neighborhood of the unit sphere tend to 1 as the size of the neighborhood shrinks to 0, then the quasisphere is said to be asymptotically conformal. While every asymptotically conformal quasisphere in n -dimensional Euclidean space has Hausdorff dimension $n-1$, in general an asymptotically conformal quasisphere may have locally infinite $(n-1)$ -dimensional Hausdorff measure.

In this talk I will describe joint work with Jim Gill, Steffen Rohde and Tatiana Toro. We obtain several Dini conditions with sharp exponent that guarantee that an asymptotically conformal quasisphere has finite Hausdorff measure. In fact our conditions guarantee that the quasisphere admits local bi-Lipschitz parameterizations. A key idea which will be discussed along the way is the connection between a good quasisymmetry constant and good approximability of an image set by hyperplanes.