NEW SOLUTION OF A PROBLEM OF KOLMOGOROV ON WIDTH ASYMPTOTICS IN HOLOMORPHIC FUNCTION SPACES

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Given a domain D in $\mathbb{C}^n$ and K a compact subset of D, we denote $\mathcal{A}_K^D$ the compact set in $C(K)$, of all restrictions in K of holomorphic functions on D bounded by 1. The sequence $(d_m(\mathcal{A}_K^D))_{m\in\mathbb{N}}$ of Kolmogorov m-widths of $\mathcal{A}_K^D$ provides a measure of the degree of compactness of the set $\mathcal{A}_K^D$ in $C(K)$ and the study of its asymptotics has a long history, essentially going back to Kolmogorov's work on ϵ-entropy of compact sets in the 1950s. The precise statement of this problem is

$$\lim_{m\to\infty} \frac{-\log d_m(\mathcal{A}_K^D)}{m^{1/n}} = 2\pi \left(\frac{n!}{C(K,D)}\right)^{1/n}, \tag{1}$$

where $C(K,D)$ is the Bedford-Taylor relative capacity of K in D. This problem has already been proved in 2004 by S.N., using pluripotential theory technics.
Here, with O. Bandtlow, we give a totally new proof of the asymptotics (1) for D strictly hyperconvex and K non-pluripolar. We proceed by a two-pronged approach establishing sharp upper and lower bounds for the Kolmogorov widths. The lower bounds follow from concentration results for the eigenvalues of a certain family of Toeplitz operators, while the upper bounds follow from an application of the Bergman-Weil formula together with an exhaustion procedure by special holomorphic polyhedra.