2024 STUDENT TALKS

Student talks for the upcoming GTA: Philadelphia 2024 conference are listed below. Talks will occur in blocks of four, roughly corresponding to the following categories:

- Algebra and Number Theory
- Topology and Homotopy Theory
- Geometric Group Theory and Dynamics
- Geometry

ALGEBRA AND NUMBER THEORY

Twisted Number Field Counting

Expository

Shilpi Mandal, *Emory University*

The archetypal question in number field counting asks that given a number field K, and a numerical invariant $inv(L/K)$ of finite extensions of K, what is the asymptotic behaviour of the counting function $N(K;X) = \#\{L/K|\ inv(L/K) < X\}$ as $X \to \infty$? Alberts proposed a 'twisted' version of Malle's conjecture, which we consider in the case of $G = D_8$ (the dihedral group of 16 elements), T a normal subgroup isomorphic to D_4, $K = \mathbb{Q}$, and M an arbitrary quadratic field. The culmination of our project would be the first non-abelian twisted count in the literature, with the Galois group acting on T by a nontrivial outer automorphism of D_4.

In this talk, I will present the twisted count and give an exposition of our approach, which is based on local-to-global principles in embedding problems. We do the twisted count by counting the number of solutions to a twisted embedding problem. Through the work on our motivating example, we have evidence that the solubility of the corresponding local embedding problems is dictated by values of Legendre symbols with modulus equal to the discriminant of certain extensions.

An Analogue of Greenberg Conjecture for CM fields

Peikai Qi, *Michigan State University*

Let K be a CM field and K^+ be the unique totally real subfield of K. Assume that primes above p in K^+ all splits in K. Let

$$\mathfrak{P}_1, \mathfrak{P}_2, \cdots, \mathfrak{P}_s, \tilde{\mathfrak{P}}_1, \tilde{\mathfrak{P}}_2, \cdots, \tilde{\mathfrak{P}}_s$$

be prime ideas in K above p, where $\tilde{\mathfrak{P}}_i$ is the complex conjugation of $\mathfrak{P}_i$. We show that there is unique $\mathbb{Z}_p$ extension of K such that $\mathfrak{P}_1, \mathfrak{P}_2, \cdots, \mathfrak{P}_s$ are totally ramified and $\tilde{\mathfrak{P}}_1, \tilde{\mathfrak{P}}_2, \cdots, \tilde{\mathfrak{P}}_s$ are unramified. We also show that such $\mathbb{Z}_p$ extension for CM field has similar properties as cyclotomic $\mathbb{Z}_p$ extension of a totally real field. We also give some criteria for Iwasawa invariant $\mu = \lambda = 0$.

Simplicial Volume Entropy of Iwahori Subgroups

Malachi Alexander, *UC Santa Cruz*

In Iwasawa theory, one is interested in studying arithmetic objects such as the ideal class group over infinite towers of number fields. We study an analogue of Iwasawa theory over non-Archimedean local fields with respect to an analogous class number defined using stable lattices (under a precompact and irreducible representation), the Bruhat-Tits building, and towers of totally ramified extensions. In previous work, Suh has shown an explicit count and bounds for the number of stable lattices in the case that the aforementioned representations have regular and irregular reduction, respectively. The goal of this work is to determine more explicit bounds for the number of stable lattices when considering a specific group representation, $\rho : I_{\vec{n}} \hookrightarrow \mathrm{GL}_K(V)$, where $I_{\vec{n}}$ is an Iwahori subgroup of $\mathrm{GL}_K(V)$ over a non-Archimedean field K for any infinite tower of finite totally ramified extensions and use the simplicial volume entropy to measure the exponential growth of the class number.

Interpolation in the Weighted Projective Space

Shahriyar Roshan-Zamir, *University of Nebraska–Lincoln*

Given a finite set of points X in the projective space over a field k one can ask for the k-vector space dimension of all degree d polynomials that vanish to order two on X. (These are polynomials whose first derivative vanishes on X.) The Alexander-Hirschowitz theorem (A-H) computes this dimension in terms of the multiplicity of the points and the k-vector space dimension of degree d monomials, with finitely many exceptions. In this talk, we investigate this question in the weighted projective line and space, $\mathbb{P}(s,t)$ and $\mathbb{P}(a,b,c)$. We define a notion of multiplicity for weighted projective spaces, give an example of $\mathbb{P}(a,b,c)$ where A-H holds with no exceptions and an infinite family where A-H fails for even one point, and discuss future directions.

Elliptic Curves, Modular Forms and Modularity Theorems

Expository

Xiaoyu Huang, *CUNY Graduate Center*

Elliptic curves are important objects in number theory and cryptography. They particularly played an important role in Taylor-Wiles proof of the Fermat's last theorem. In the proof, they proved the modularity theorem, making a connection between elliptic curves and some mysterious objects: modular forms. Analogous conjectures are expected for varieties of higher dimensions and automorphic form in the Langlands program. In this talk, I will explain the connection and its implication for modern mathematics.

Admissibility over Number Fields

Deependra Singh, *University of Pennsylvania*

Given a field K, one can ask which finite groups G are Galois groups of field extensions L/K such that L is a maximal subfield of a division algebra with center K. Such a group G is called admissible over K. Like the inverse Galois problem, the question remains open in general. But unlike the inverse Galois problem, the groups that occur in this fashion are generally quite restricted. In this talk, I will discuss some results over number fields, including an extension of J. Sonn's result about admissibility of solvable Sylow-metacyclic groups over the rationals.

Integral Points on Varieties with Infinite Etale Fundamental Groups

Niven Achenjang, *MIT*

One natural and long-studied question is, "Given a system of polynomial equations, can one describe its integer solutions?" In more modern parlance, given a variety/scheme $U/\mathbb{Z}$, describe $U(\mathbb{Z})$. For example, is it finite or infinite? In practice, one often takes U to be the complement $X - D$ of a divisor D in a projective variety X. For proving finiteness of such $U(\mathbb{Z})$, it is usually advantageous to choose D with many irreducible components. In this talk, after summarizing the basics of integral points, I will present a fairly general construction producing examples of varieties $U = X - D$ of the previously described form where D is irreducible, but $U(\mathbb{Z})$ is nevertheless provably finite. This is joint work with Jackson Morrow.

GEOMETRIC GROUP THEORY AND DYNAMICS

Large-Scale Geometry of Pure Mapping Class Groups of Infinite-Type Surfaces

Thomas Hill, *University of Utah*

The work of Mann and Rafi gives a classification of surfaces Σ when $\mathrm{Map}(\Sigma)$ is globally CB, locally CB, and CB generated under the technical assumption of tameness. In this article, we restrict our study to the pure mapping class group and give a complete classification without additional assumptions. In stark contrast with the rich class of examples of Mann--Rafi, we prove that $\mathrm{PMap}(\Sigma)$ is globally CB if and only if Σ is the Loch Ness monster surface, and locally CB or CB generated if and only if Σ has finitely many ends and is not a Loch Ness monster surface with (nonzero) punctures.

Dehn Functions

Expository

Eleanor Rhoads, *Wesleyan University*

For a finitely presented group with a solvable word problem, a natural question to ask is how hard it is to actually solve the word problem. One tool we have in addressing this question is to examine the group's Dehn function, which describes how much "area" a trivial word can capture relative to its word length. In this talk, we'll define the Dehn function for a group, and interpret what they tell us about both the group's algebraic structure and its geometric structure. Lastly, we'll discuss the Dehn function for some important classes of groups in geometric group theory.

A Long Exact Sequence on the Composition of Dehn Twists

Shuo Zhang, *University of Minnesota Twin Cities*

Seidel conjectured that the fix-point-Floer homology of the global monodromy (iterated Dehn twists) of a Lefschetz fibration can be computed from the Lagrangian Floer homologies of the vanishing cycles. In particular there should be a long exact sequence relating them. We construct this long exact sequence by iterating Mak-Wu's construction of a Lagrangian cobordism of a Lagrangian correspondence associated. Potential applications include homological mirror symmetry for Fano manifolds and dynamics of iterated Dehn twists.

Hyperbolic Groupoids with Totally Disconnected Unit Spaces and Their Associated Semigroups

Josiah Owens, *Texas A&M University*

Hyperbolic groupoids generalize the notions of Gromov hyperbolic groups, hyperbolic dynamical systems, and Ruelle-Smale spaces. I will provide a very brief introduction to the theory of hyperbolic groupoids, highlighting their most notable features, their utility, and their hyperbolic duality, providing several examples along the way. Then I will discuss my present research on the class of hyperbolic groupoids with totally disconnected unit spaces and their associated semigroups which algebraically encode the geometric and topological features of the groupoid in such a way that it may be reconstructed from its associated semigroup; I also provide a definition for the class of abstract semigroups which may be used to construct hyperbolic groupoids. This theory is a work in progress.

Symplectic Billiard Table Evolution

Lael Costa, *Penn State University*

The symplectic inner billiard, introduced by Albers and Tabachnikov, is an affine-invariant relative of the usual inner billiards. Like the usual billiard, it is played inside a fixed convex "table," and a natural question to ask is, which periodic orbits exist for a given table? In this talk, I will discuss the inverse problem: given a closed trajectory of the symplectic billiard map, what polygonal table could have produced that orbit? Iterating this process gives a dynamical system on affine equivalence classes of convex polygons, expressed through a simple geometrical construction. I will demonstrate preliminary theoretical results as well as my computer experiments.

Arithmeticity, Bounded Distance, and Coarse Geometry—All About Quasi-isometric Rigidity

Expository

Yushan Jiang, *CUNY Graduate Center*

When we consider a finitely generated group, we can look at the algebraic and (coarsely) geometric properties of it. The celebrated Gromov's theorem on groups of polynomial growth tells us there are very deep relationships between these two kinds of properties. In this talk, I will introduce quasi-isometric rigidity of groups which follows Gromov's program: what can we figure out by observing its coarse geometry.

In particular, I will focus on the fantastic results obtained by R. Schwartz about rank 1 non-uniform lattices (e.g. fundamental groups of non-compact complete hyperbolic 3-manifolds with finite volume): quasi-isometry implies commensurability. In addition, combining with A. Reid's famous work on arithmetic knot complement, one can see the uniqueness of the figure 8 knot from the coarsely geometric point of view.

Orientation-Preserving Homeomorphisms of Euclidean Space Are Commutators

Megha Bhat, *CUNY Graduate Center*

A uniformly perfect group has commutator width p if every element can be expressed as a product of p commutators. Questions about commutator width have been asked and answered for various groups such as the alternating group and the symmetric group. I will talk about this question for homeomorphism groups of spheres, annuli and Euclidean space, and show that each of these has commutator width one.

Equivariant Trees and Partition Complexes

Maxine Calle, *University of Pennsylvania*

Given a finite set, the collection of partitions of this set forms a poset category under the coarsening relation. This category is directly related to a space of trees, which in turn has interesting connections to operads. But what if the finite set comes equipped with a group action? What is an "equivariant partition"? And what connection is there to equivariant trees and equivariant operads? We will explore possible answers to these questions in this talk, based on joint work with J. Bergner, P. Bonventre, D. Chan, and M. Sarazola.

Random Walks on Ramanujan Graphs

Expository

Amin Idelhaj, *University of Wisconsin–Madison*

I will discuss the proof that random walk on Ramanujan graphs exhibits the "cutoff phenomenon". Time permitting, I will show how ideas in this proof relate to the representation theory of $PGL_2(\mathbb{Q}_p)$.

Profinite Rigidity

Expository

Zihao Liu, *Rice University*

Profinite rigidity is the study of the extent to which a group is determined by its set of finite quotients, or, in more sophisticated terminology, by its profinite completion. In this talk, I will give a brief overview of what is known about this in the context of groups arising in low-dimensional topology and geometry.

TOPOLOGY AND HOMOTOPY THEORY

What Is an Alternating Link?

Expository

Susan Rutter, *CUNY Graduate Center*

Intuitively, a link is alternating if it admits a diagram where each component's crossings alternate between over and under crossings. In 2015, Greene introduced an alternative characterisation in terms of the Gordon Litherland pairing, allowing for a new proof of the Tait conjecture that all reduced alternating diagrams have the same crossing number and writhe. I will present this paper.

Hochschild Homology of the Legendrian Contact DGA

Alexander Simons, *UC Davis*

Given a positive braid β, there is a natural way to take the closure and obtain a Legendrian knot. We prove that the associated Legendrian contact dg-algebra $\mathcal{A}_\beta$ is a resolution of the augmentation variety $H_0(\mathcal{A}_\beta)$. This allows us to compute derived invariants such as Hochschild homology, which has implications for Stein traces and derived algebraic geometry.

Using Quandle Invariants to Distinguish Classical and Legendrian Knots

Expository

Peyton Wood, *UC Davis*

We will define quandles, show why these are natural algebraic objects to consider when studying oriented knots, and examine how they can be used to classify knots. We will specifically review the fundamental quandle (also called the knot quandle), quandle counting invariant, and quandle State-Sum Invariant. For the state-sum invariant, we will discuss emerging strategies for using this invariant to tabulate prime knots and distinguish between mirror images when possible. Then, we will give a brief introduction to Legendrian knots, define Legendrian racks (and quandles) via the front projection and show how to use these racks (and quandles) to distinguish Legendrian knots using a similar counting invariant and State-Sum Invariant.

Topological Obstructions to the Existence of a Resolution of the Singularities of a Variety over a Field of Any Characteristic

James Myer, *CUNY Graduate Center*

In 1947, Norman Steenrod asks "When can a cycle in the homology of a space represented by a manifold?" In 1954, René Thom develops topological obstructions to the representation of a cycle by a manifold, and proves affirmation when the dimension of the space is at most six, yet leverages these obstructions to provide a startling refutation when the dimension is at least seven, exhibiting an explicit counterexample.

Years later (2004), Dennis Sullivan suggests the famous question of whether every variety admits a resolution of its singularities motivated Thom's attack on Steenrod's Problem. Only 10 years after Thom's attack does Heisuke Hironaka prove every variety over a field of characteristic zero admits a resolution of its singularities.

We still do not know whether every variety over a field of positive characteristic enjoys a resolution of its singularities. Sullivan also suggests in 2004 that Thom's strategy can shed light on this problem. So, we realize Sullivan's vision, and construct topological obstructions to the existence of a resolution of the singularities of a variety over a field of positive characteristic à la Thom.

Juggling Virtual Braids and Links

Ivy Stump, *Davidson College; Temple University*

Have you ever wondered how to think about juggling tricks as topological objects? In this talk, we establish a relationship between juggling patterns and virtual links: oriented links in 2-space that permit positive, negative, and virtual crossings. To do this, we extend Satyan Devadoss and John Mugno's results of mappings onto solid torus braids for two-handed juggling. We accomplish this by defining what constitutes an n-handed braid diagram for juggling patterns, as well as a function $\mathcal{J}_n$ onto abstract links. We find that $\mathcal{J}_n$ is a surjection via an immersion in $\mathbb{R}^2$. Due to the properties of virtual links as diagrams on surfaces of arbitrary genus or alternatively as Gauss codes, this direction invites new and exciting research opportunities at the juxtaposition of mathematical juggling and low-dimensional topology.

Categorization of Finite Field Polygons, and Why Graph Arguments Are Cooler than Algebra Ones

Amanda Tran, *Tufts University*

Every complex of lines in $\mathbb{P}(\mathbb{Z}_2^4)$ can be represented as an incident matrix, where a matrix without full rank signifies to us the presence of a linear redundancy, which can be recovered matrix wise, polynomial wise, as well as by visual inspection of the collection's drawing (there's a modified graph coloring argument here).

However, that's not fun enough! I'm not an algebraic geometer, I'm a graph theorist at heart! Ultimately, we want a "make money quick" way of checking for linear redundancies and types *without* having to color things in by hand (expensive), RREF-ing the bejeezus out of everything (boring), or hunting for more polynomials (pandering).

In this project, we seek to (1) classify minimal linear combinations, (2) associate with each a name, shape, polynomial, and matrix, (3) develop a nice forbidden subgraph argument (perhaps a counting one?)

GEOMETRY

Deformation Spaces of Geometric Structures

Expository

Gabe Lumpkin, *George Mason University*

The study of (G, X)-structures on manifolds is concerned with endowing a manifold with a local geometric structure modelled on some homogeneous space X. In this case, X is a simply connected smooth manifold and G is its group of isometries. The same topological surface can be given uncountably many distinct geometric structures, which gives rise to the study of *deformation spaces* - topological spaces whose points parametrize geometric structures on a given surface. The classical example of such a space is the *Teichmüller space* of hyperbolic structures on a closed genus $g \geq 2$ surface. The deformation space of geometric structures on a manifold M can be locally modeled by the *character variety*, $Hom(\pi_1(M), G)/G$, via the *holonomy map*. This allows one to give a concrete description of the deformation space, which will in turn have interesting topology and geometry.

In this talk, we will introduce geometric structures on manifolds, study some examples, and consider the relationship between the deformation space and the character variety. We will see some examples of character varieties, and, in particular, we will see why the ordinary conjugation quotient is unsatisfactory and how one can compute the *GIT quotient*.

Stability of Convex Spheres

Hunter Stufflebeam, *University of Pennsylvania*

We prove that strictly convex 2-spheres, all of whose simple closed geodesics are close in length to 2π, are C^0 Cheeger-Gromov close to the round sphere.

Realizing Pairs of Multicurves as Cylinders on Translation Surfaces

Juliet Aygun, *Cornell University*

A translation surface is a Riemann surface equipped with a holomorphic differential. Equivalently, a translation surface is a collection of polygons in the Euclidean plane in which each side is identified with another by translation. A pair of intersecting curves on a topological surface is called coherent if they can be oriented so that their geometric and algebraic intersection numbers are equal. In this talk, we investigate when a pair of multicurves on some topological surface can be simultaneously realized as the core curves of Euclidean cylinders on some translation surface. This is joint work with Janet Barkdoll, Aaron Calderon, Jenavie Lorman, and Theodore Sandstom.

Sharpness in Symplectic Topology

Spencer Cattalani, *Stony Brook University*

Among all almost complex manifolds, those which are tamed by symplectic forms are particularly well studied. What geometric properties characterize this class of manifolds? I will explain two recent results in this direction, which confirm speculations posed by M. Gromov in 2000 and 1985, respectively. The first is that an almost complex manifold admits a taming symplectic structure if and only if it satisfies a certain bound on the areas of coarsely holomorphic curves. The second is that an almost complex 4-manifold which has many pseudoholomorphic curves admits a taming symplectic structure. This leads to an almost complex analogue of D. McDuff's classification of rational symplectic 4-manifolds.

Flexing and Branched Bending

Casandra Monroe, *The University of Texas at Austin*

For a hyperbolic n-manifold, bending along a totally geodesic hypersurface is a well-studied method of producing deformations of the original hyperbolic structure. However, there are instances where certain computations show that deformations of a hyperbolic structure exist, even in the absence of totally geodesic hypersurfaces to bend along. In a paper of Cooper, Long, and Thistlethwaite, a manifold is called "flexible" if it has this property. But what explains when a manifold is flexible? In this talk, we will explore one potential answer: a generalization of bending, where instead we bend along totally geodesic branched complexes.

Half-Spaces & Einstein's Universe

Pier-Olivier Rodrigue, *George Mason University*

The Teichmüller space of a surface can be viewed as a subset of the moduli space of representations of the surface's fundamental group into the group $\mathrm{PSL}(2, \mathbb{R})$. The space of maximal representations of the fundamental group of a surface generalizes Teichmüller spaces by replacing the group $\mathrm{PSL}(2, \mathbb{R})$ with another Lie group of Hermitian type such as $\mathrm{Sp}(2n, \mathbb{R})$, $\mathrm{SU}(p, q)$, or $\mathrm{SO}(2, n)$.

In my master's thesis, we established a construction of hypersurfaces in the space of photons in Einstein's universe, a homogeneous space of $\mathrm{SO}(2, n)$. The definition of these hypersurfaces simplifies the construction of disjoint half-spaces. These half-spaces, in turn, can be utilized to create fundamental domains for certain discrete subgroups of the indefinite orthogonal group $\mathrm{SO}(2, n)$, including, among others, the images of maximal representations of surfaces with boundaries.

This talk will primarily focus on the construction of these half-spaces. We will begin by examining the construction of Fuchsian Schottky Groups and how it was generalized by Burelle and Treib, and finally adapt their construction to the Einstein Universe.

A Foliation on the Space of Framed Hyperbolic 3-Manifolds

Matthew Zevenbergen, *Boston College*

Two infinite volume framed hyperbolic 3-manifolds are close in the Chabauty topology, also known as the geometric topology, if their geometry is similar on large neighborhoods of the base framing. There is a natural "almost"-foliation on this space, each leaf of which corresponds to the set of possible framings on a fixed hyperbolic 3-manifold. We will study the topology of the leaves, and construct a leaf (corresponding to a particular non-tame hyperbolic 3-manifold) which is a dense set of the Chabauty topology.

On the Space of Holomorphic Differentials

Margarita Bustos Gonzalez, *University of Iowa*

Investigating the space of holomorphic differentials is a problem that originated by the German mathematician Erich Hecke. Currently, this is still an open problem. We begin with a smooth projective curve X over an algebraically closed field k of characteristic 2. Assume that X has a right faithful action by the alternating group on 4 letters A_4. This induces a left group action on its space of holomorphic differentials. Hence, the space has a kA_4-module structure. We would like to find the precise module structure of the space as a direct sum of indecomposable kA_4-modules by using some concepts of number theory, Kummer theory, and Artin-Schreier theory.

Contractibility of Teichmüller Space

Expository

Chaitanya Tappu, *Cornell University*

In this talk, we sketch a proof of the fact that the Teichmüller space of a Riemann surface X is contractible. This proof works for any Riemann surface, whether it is of finite type or of infinite type. The proof uses a theorem of Douady and Earle which provides the construction of a conformally natural extension of circle homeomorphisms to homeomorphisms of the disk.

The Hot Spots Conjecture with Mixed Boundary Conditions

Lawford Hatcher, *Indiana University*

The hot spots conjecture of J. Rauch states that the second Neumann eigenfunction of the Laplace operator on some bounded domain takes its extrema only on the boundary of the domain. This conjecture is open in full generality, and the location of the extrema is intimately and mysteriously related to the geometry of the domain. We will present a variant of the problem using mixed Dirichlet-Neumann boundary conditions. After discussing the relationship between these two problems, we will present some partial results on the latter problem.